Significant figures

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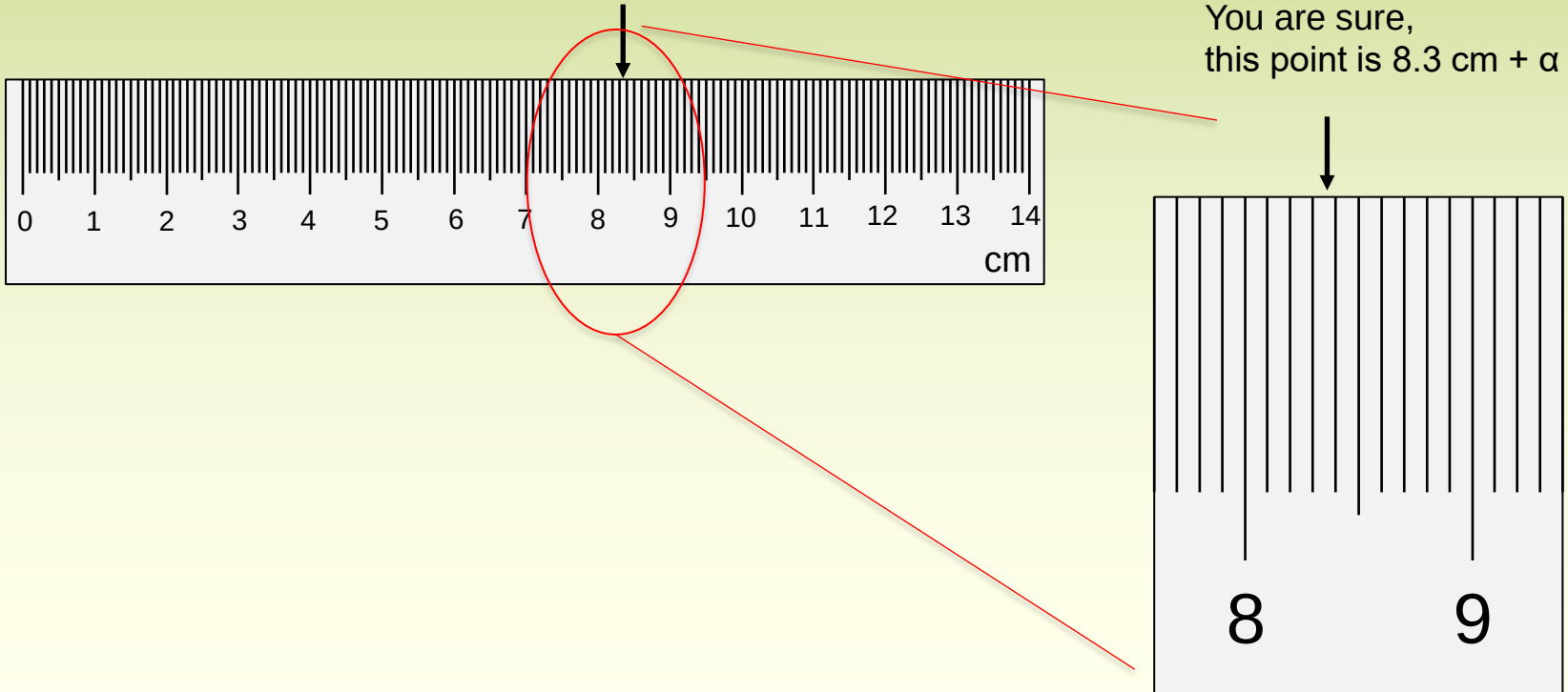
Significant Figures

- There is always an error in the measurement
  - Some of you may learn Probability and Statistics in high school
  - Two type of errors: Statistical and Systematic Error
- In this experimental course, “Accuracy = Significant figures” are emphasized
- Spreadsheet application (e.g. Excel, Numbers)
  - By default setting, if zero continues in the digits after the decimal point, it will be erased without permission.
  - There is NO setting of “significant figures” even if “the number of digits after the decimal point” is adjusted
  - You need to set the number of digits in EACH cell with considering the significant figures for the value

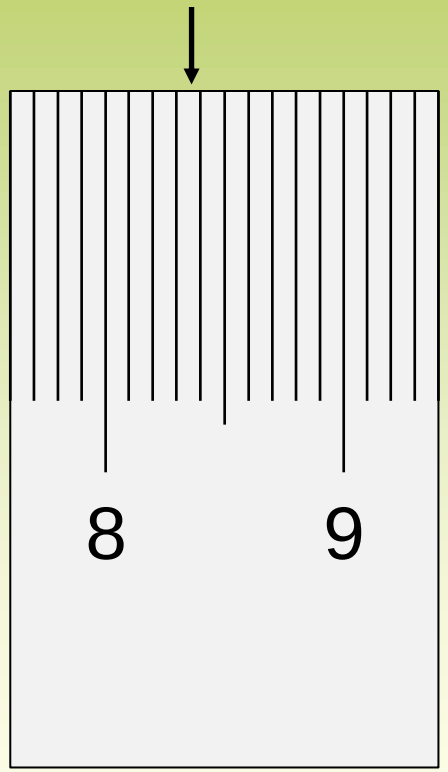
Accuracy

- Example: Ruler
  - You may read 1/10 value of the smallest scale



Accuracy

You are sure,
this point is at least more than 8.3 cm and less than 8.4 cm.



If you say, it is “8.35 cm”

The last digit have uncertainty.
8.35 means
more than 8.34 and less than 8.36

Accuracy of Rounded Numbers

- 1.0 and 1.00 are NOT the same
  - If x = 1.0
    - The range of x: $0.95 \leqq x < 1.05$
  - If x = 1.00
    - The range of x: $0.995 \leqq x < 1.005$
- In case of 8.35 and 8.350
  - x = 8.35
    - $8.345 \leqq x < 8.355$
  - x = 8.350
    - $0.83495 \leqq x < 8.3505$

Using the Supplementary Unit

- 8.35 cm is same value with
  - 0.0835 m
  - 0.0000835 km
  - 83500 μm

This expression is WRONG in scientific manner

- You should keep the significant figures with any order of supplementary unit

Using the Supplementary Unit

- 8.35 cm is same value with
  - 8.35×10^{-2} m
  - 8.35×10^{-5} km
  - 8.35×10^{4} μm

- It is better to use a power of 10
  - It is clear to recognize the significant figures

Calculation with Measured Values

- In the calculation, you need to take care the significant figures of measured values
  - For example, area of circle: S
    - The radius r = 8.35 cm
    - $S = \pi \times r^2 = 3.14159265358979..... \times (8.35)^2$

      $= 219.03969378991413327 5.....$

Do you think
it makes sense to have
too many digits?

Calculation with Measured Values

- The radius r = 8.35 cm
  - Means: 8.345 cm $\leqq$ r < 8.355 cm
  - $S = \pi \times r^2$
    - more than or equal to $\pi \times (8.345)^2 = 218.7777449\ldots$
    - less than $\pi \times (8.355)^2 = 219.302095\ldots$
  - Meaningful digits are 3 digits
    - S = 219 cm^2
- In this examples is the case of multiplication
  - How about the other case?
  - Let's see the example from the subject 4 "Electrical resistivity"

Example

- In case of addition and subtraction, consider where the digits are accurate

Table shown in the textbook

I (mA)	V_+ (mV)	V_- (mV)	$V = (V_+ - V_-)/2$ (mV)	$R = V/I$ (Ω)
0.00	0.002	0.002	0.000	-
2.00	0.049	-0.046	0.048	0.0237
4.00	0.097	-0.093	0.095	0.0238
⋮	⋮	⋮	⋮	⋮
18.00	0.430	-0.427	0.429	0.0238

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2 digits of
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"2" is integer to take the average. That means that zeros continue infinitely after the decimal point. Infinite number of significant figures

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2 digits of significant figures

$$(0.049 - (-0.046))/2 = 0.0475$$

8

“2” is integer to take the average. That means that zeros continue infinitely after the decimal point. Infinite number of significant figures

Significant digits are 3 digits after the decimal point

Example

- In the case of multiplication and division
  - Adjust to the one with the smallest significant figures

Table shown in the textbook

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2 digits of Significant figures

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3 digits of
significant figures

2 digits of
Significant figures

After the division
Significant figures are 2 digits

Correct number is 0.024

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4 digits of
significant figures

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- In the case of multiplication and division
  - Adjust to the one with the smallest significant figures

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⋮	⋮	⋮	⋮	⋮
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4 digits of significant figures

3 digits of significant figures

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OK

4 digits of significant figures

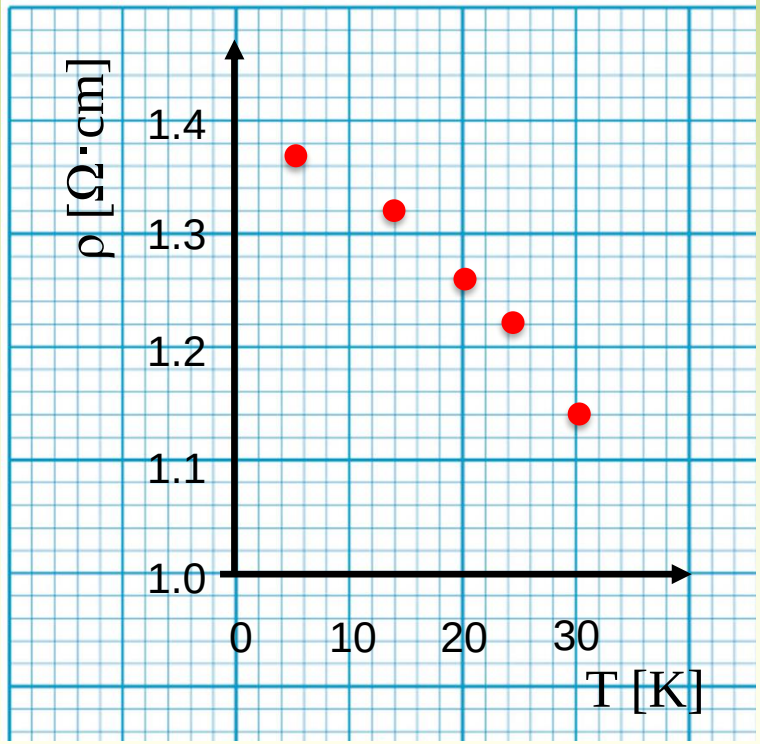
3 digits of significant figures

Division of 4 digits and 3 digits of significant figures

Significant of figures of the result is 3 digits

Significant figures and Graph

- Calculation of the resistivity
  - Thickness, width, radius, length: the smallest of significant figures is 2 digits
  - Even if significant figures is 4 or 5 digits, significant figures of resistivity is 2 digits
  - On the other hand, If you use 2 digits to make a graph, the distribution may become not smooth
    - Method A: Use the values before rounding for plot



For example,
The result of calculation is 1.37
Significant figures is 2 digits
The meaningful value is 1.4

T [K]	ρ [Ω·cm]
5.5	1.37
12.0	1.32
20.5	1.26
24.5	1.22
30.5	1.14

Significant figures and Graph

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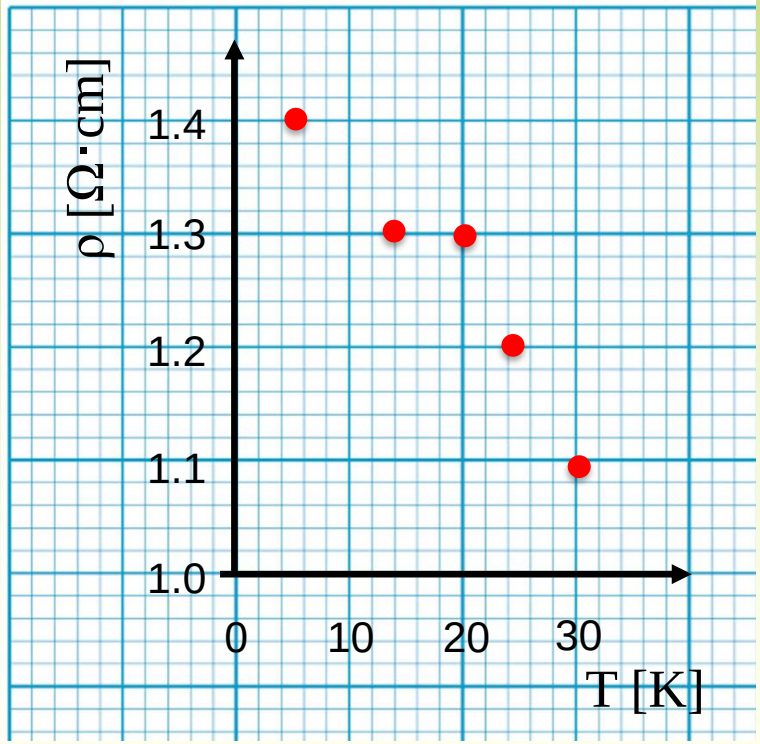
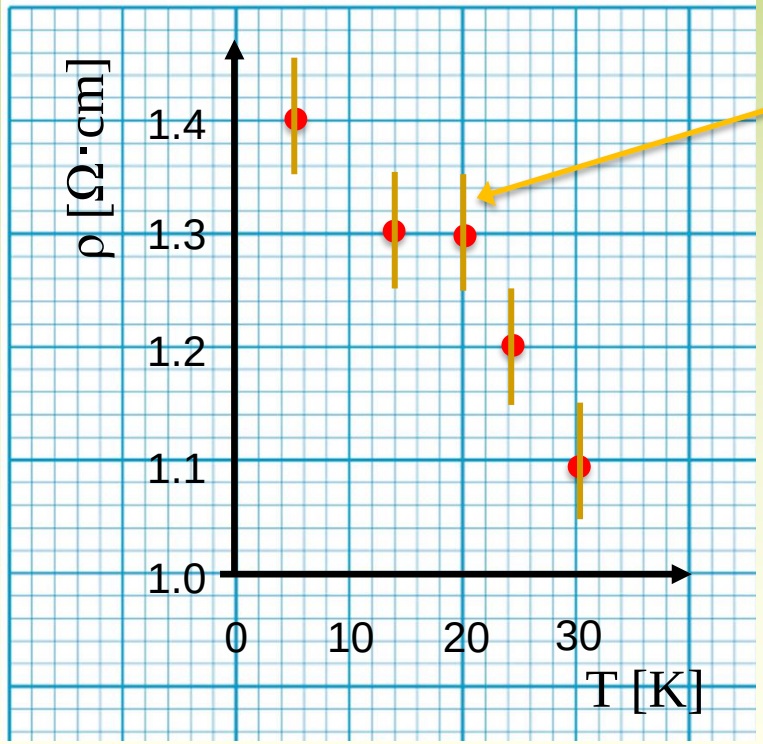


T [K]	ρ [Ω·cm]
5.5	1.4
12.0	1.3
20.5	1.3
24.5	1.2
30.5	1.1

Significant figures and Graph

- Calculation of the resistivity
  - Thickness, width, radius, length: the smallest of significant figures is 2 digits
  - Even if significant figures is 4 or 5 digits, significant figures of resistivity is 2 digits
  - On the other hand, If you use 2 digits to make a graph, the distribution may become not smooth
    - Method B: Use the values rounded with error bars that show the range of uncertainty



Range of uncertainty

T [K]	ρ [Ω·cm]
5.5	1.4
12.0	1.3
20.5	1.3
24.5	1.2
30.5	1.1

Significant figures and Graph

- When you plot graphs
  - Always consider significant figures of the values
  - You can use either solution A or B
    - Please mention the reason why you select which method
    - For example
      - “The significant figures is 2 digits of the resistivity. But if I use the digits the graph, the data points will be discrete and will have a ratting distribution. Therefore, I used 3 digits to make the graph of the resistivity as a function of the absolute temperature.”
    - The important point is that you need to present what you understood about significant figures